Geometry for Real People – Points, Lines and Planes

Geometry is a study of descriptions. It is the mathematical method for describing shapes and spatial relationships. Geometry begins with a network of definitions that are then combined to make other definitions that are then used in proofs of even more definitions. In this article, I will focus on the simplest definitions and relationships: those that deal with Points, Lines, and Planes.

Postulate versus Theorem

Both Postulates [also called Axioms] and Theorems define relationships. We couldn't get anywhere with geometry if there were not SOME things we could agree were true without having to prove them mathematically. Geometry does involve proving theorems, of course, but I will leave that to other discussions. I love proofs, but most sane people do not. This article describes the logic of Postulates and Theorems to help you understand proofs and work with them more easily.

Point

A Point is a specific position in space. A point has no dimensions; it is just a position. In either a drawing or a mathematical statement, a dot and one capital letter represent a point. For example, in Figure 1 A, B, and C are all points.

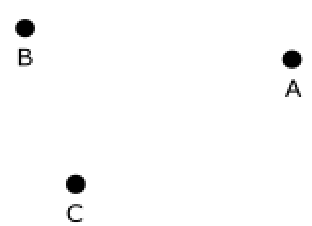

Figure 1

Line

A line is a continuous set of points that extend in opposite directions from a given point for an infinite distance. In mathematical statements a line is identified by naming two points on the line and usually drawing the symbol ↔ over the letters that represent the two points. This convention can be seen in figure 2. For text simplicity this article will use ←AB→ as needed. Three lines are shown in Figure 2: ←AB→, ←AC→, and ←BC→. Points that lie on the same line are referred to as *collinear*.

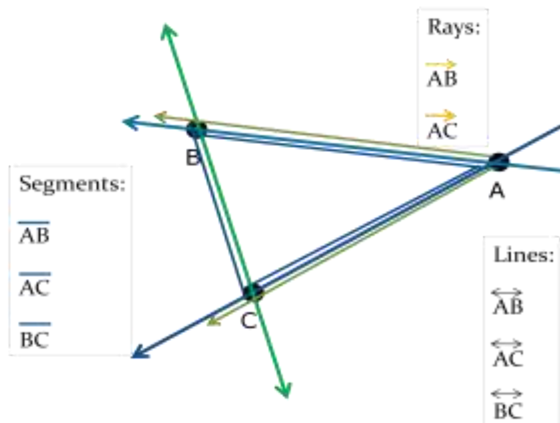

Figure 2

Line Segment

A line segment is a piece of a line that ends at two specific points. To identify a line segment, one can write AB with or without a line over it, as you can see in Figure 2. The points on each end of the line segment are referred to as its *endpoints*.

Ray

A ray is a portion of a line that consists of a specific point (the endpoint) and the set of all points on one side of the endpoint. To indicate on WHICH side of the endpoint the rest of the points lie, it is usual to designate another point along the ray. This way the ray can be written AB→ or with the arrow above the letters, as shown in Figure 2.

Line Postulate 1 - The Ruler Postulate

1. **Any two points on a line may be assigned the values 0 and 1. Using these two points as a gauge, all other real numbers can then be represented on the line.** Here's the idea: there are an INFINITE number of points on any line [remember points have no dimensions]. You pick any two and call them 0 and 1, and then you know how far apart are the points on the line representing ALL OTHER integers. This process is referred to as establishing a *coordinate system*.
2. **Once the coordinate system, above, is in place, the distance between two points on the line equals the absolute value of the difference between their coordinates.** Basically, the distance between any two points [in the units described in #1 above] can be calculated by subtracting one coordinate from the other and taking the absolute value of the answer.

Line Postulate 2 - **Through any two points there is exactly one line**. You can't 'fit' more than one line, because, as we know, a point has no dimensions, JUST position!

Line Postulate 3 - The Line/Plane/Space Postulate

1. **A line contains at least 2 points**. (But then, you knew that already!)
2. **A plane contains [or can be defined by] three points not all on the same line**. Look at it this way: a line has only one dimension, length. THINK about that for a moment. All of the points lie one next to the other, and even though the points themselves have no dimensions, they do describe a LENGTH through the universe. (No width, right?) No matter how many of them there are on that line, their very 'skinniness' means that the line can never have any width. It will take another point, one that is NOT on the line, to produce a plane [like a flat piece of paper], that is, to provide some width. Points or lines that lie in the same plane are referred to as *coplanar*.
3. **Space contains at least 4 points not all in one plane.** If you understood statement #2, this one involves a very similar leap of logic. A plane has just TWO dimensions – length and width. The points that make it up are still 'skinny' and STILL can't give it a third dimension. It takes yet ANOTHER point, one NOT in the plane, to impart the third dimension, depth.

Line Theorem 1 – If two lines intersect, they meet at EXACTLY one point. Sure! You have this one; the lines are too 'skinny' to overlap in more than JUST ONE point.

Line Segment Postulate 1 – If point B is between points A and C on a line, then the following is true:

$$AB + BC = AC$$

Figure 3

Line Segment Theorem 1 – The Midpoint Theorem – If B is the midpoint of line segment AC, then is true that:

$$AB = BC$$
$$AB = \frac{1}{2} AC$$
$$BC = \frac{1}{2} AC$$

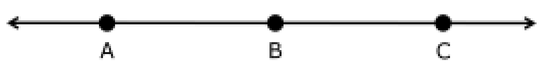

Figure 4

Plane Postulate 1 –

1. **Through any three points there is at least one plane.** You know this from the Postulate above that defined a plane.

2. **Through any three NONCOLLINEAR points there is exactly one plane.** This is here to remind you [by stating the opposite] that if ALL three points are on the same line, there are an infinite number of planes that contain them [in any and all positions in the third dimension]. Figure 5 shows just two planes out of the infinite number that intersect in space in a single line. A third point is needed to define EXACTLY which one of that infinite number of planes it is.

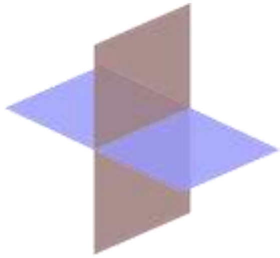

Figure 5

Plane Theorem 1 – Through a line and a point not on the line there is exactly ONE plane. You can see this one by remembering that if there are three points, there will ALWAYS be exactly one line that includes at least two of them. This involves the same logic as in #2, above.

Plane Postulate 2 – If two points are in a plane, then the line between them is in that plane. The line has to go straight between the two points, and none of them, the points, the plane, nor the line, has any depth, so the line must lie in the plane.

Plane Postulate 3 – If two planes intersect, then their intersection must be a line. You can see that this must be true from Figure 5. Keep in mind that a plane, similar to a line, continues infinitely in BOTH of its dimensions. No two planes can 'touch' except in a line.

Plane Theorem 2 – If two lines intersect, then exactly one plane contains both lines. Consider Figure 6. You know from postulates above that because they are on the same line, points A and B are in a single plane, because every point on a single line lies in the same plane. Likewise, points C and D are in a single plane. If the two lines intersect at point E, then point E is collinear with ALL of points A, B, C, and D. Therefore, all of the points and both lines lie in a single plane.

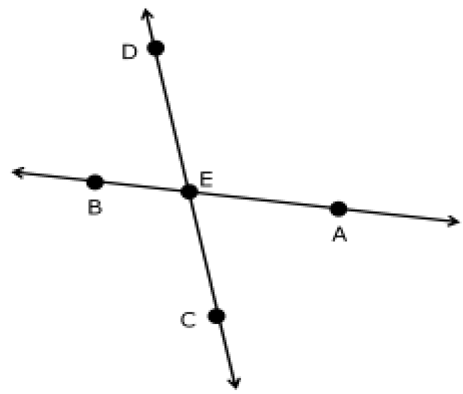

Figure 6

A Final Note: Every textbook writer numbers the postulates and theorems differently. I have tried to give them vaguely descriptive names wherever possible. I have to leave it to you to apply the appropriate numbering from your text to my explanations,